Homework 5 Problems:

(For 12 or 13th edition) Section 16.5: 34, 36, 44, 46, 48, 50, 52, 54

1. Let A be a matrix of size 3 × 2. Show that $|A(e_1) \times A(e_2)| = \sqrt{\det(A^T A)}$.
2. Let $\mathbb{C}$ be the set of all complex numbers and let $\mathbb{C}^2$ be the set $\{(z, w) | z, w \in \mathbb{C}\}$. Find the area of the part of the surface $w = z^2$ in $\mathbb{C}^2$ where $|z| \leq 1$.
3. Find the three-dimensional volume of the unit sphere $\{x \in \mathbb{R}^4 | |x| = 1\}$ in $\mathbb{R}^4$. (To be continued)
4. Let M be the set of all 2 × 2 matrices A with rank less than or equal to 1 and $|A| \leq 1$. Find the three-dimensional volume of M (here $|A|^2$ = the sum of squares of the entries of A).